Math 75 Exam 1 Name: My Key.

Fall 2014 Version A

Section (circle one): Arnold (12:00 and 1:00) Kelm (11:00) de Souza (10:00)

Instructions.

- ☐ Write your full name and circle your section time above.
- ☐ Please write the following information on your Scantron form:
  - { Under NAME, write your full name.
  - { Under SUBJECT, write your section time (10:00, 11:00, 12:00 or 1:00).
  - { Under TEST NO., write \A".
- ☐ Your answers to the multiple choice questions must be marked on your Scantron form (and nothing written in your exam booklet will be considered for the multiple choice questions). Use the space provided in the booklet to complete the free response questions.
- ☐ Please make sure to double-check your answers.
- ☐ No calculators, no cellphones, or notes are allowed on this exam. All electronic devices must be silent and stowed.

Multiple choice questions

1. Find the vertical asymptote(s) for the graph of the function $f(x) = \dfrac{x^2 + 3x + 2}{x^2 + 4x + 3}$.

(a) $x = 0$

(b) $x = -3$

(c) $x = -2$ and $x = -1$

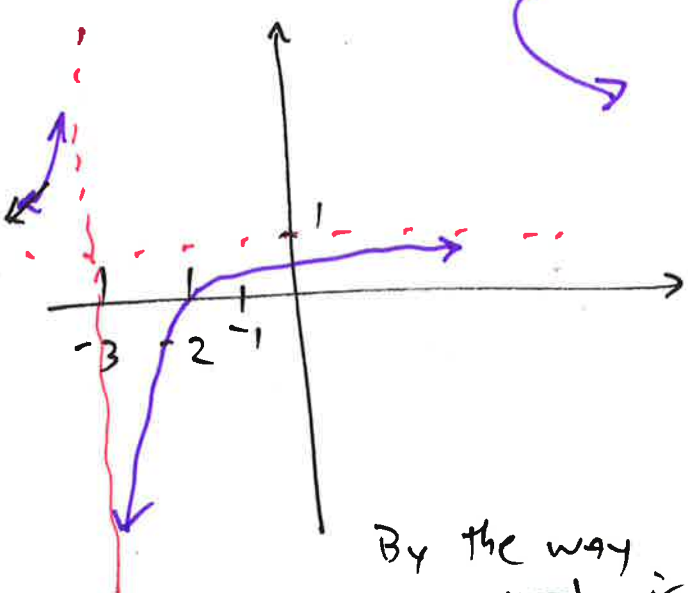
(d) $x = -3$ and $x = -1$

(e) $x = -1$

$$\frac{(x+2)(x+1)}{(x+3)(x+1)}$$

$x = -3$ v.assymp.

$x = -1$ removable discontinuity.



By the way $x = 1$ is horizontal assymp.

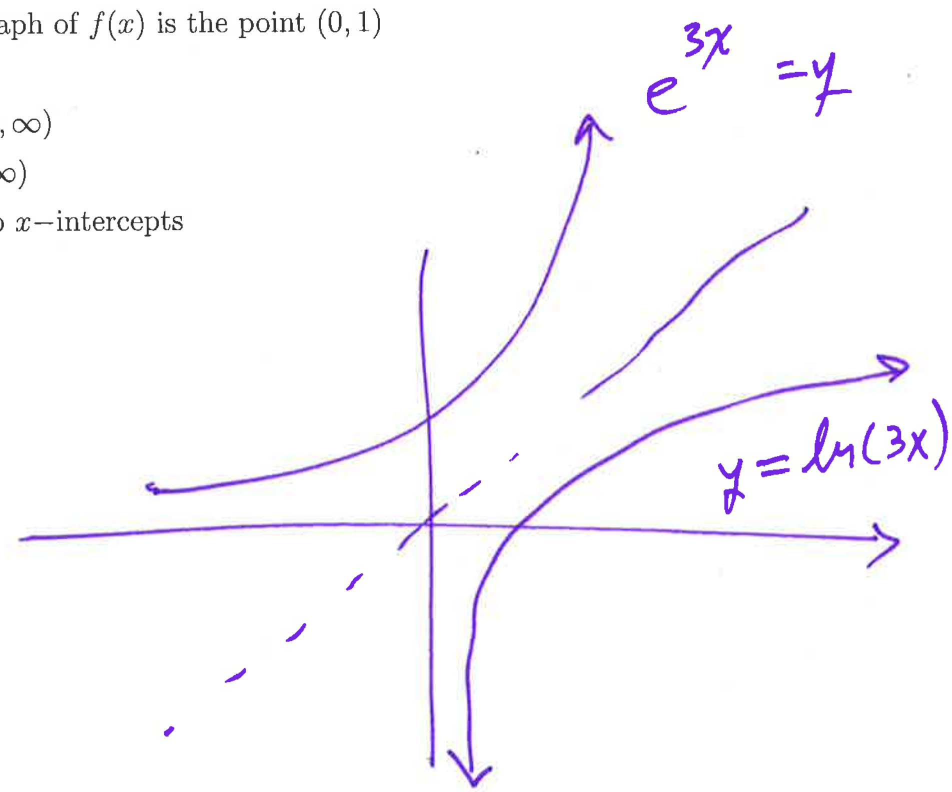
2. If $h(x) = (f \circ g)(x)$, $f(1) = 2$, $f(2) = 5$, $f(3) = 10$, $f(4) = -10$, $g(1) = 4$, $g(2) = -7$, $g(3) = -4$, $g(4) = 13$ find $h(1)$.

   (a) -10
   (b) 10
   (c) 5
   (d) -7
   (e) 13

$h(1) = f(g(1)) = f(4) = -10$

3. Which of the following statements is true about the function $f(x) = \ln(3x)$?

   (a) The y-intercept of the graph of $f(x)$ is the point $(0, 1)$
   (b) $f(x)$ is decreasing
   (c) The domain of $f(x)$ is $(0, \infty)$
   (d) The range of $f(x)$ is $(0, \infty)$
   (e) The graph of $f(x)$ has no x-intercepts



4. $\lim_{x \to 9} \dfrac{\sqrt{x}-3}{x-9} = \ldots$

(a) ∞

(b) This limit does not exist

(c) $\dfrac{1}{6}$

(d) $\dfrac{1}{3}$

(e) $\dfrac{1}{9}$

$$\frac{(\sqrt{x}-3)\cdot 1}{(\sqrt{x}-3)(\sqrt{x}+3)}$$

$$\lim_{x\to 9} \frac{1}{\sqrt{x}+3} = \frac{1}{\sqrt{9}+3} = \frac{1}{6}$$

5. The inverse of the function $f(x) = \sqrt{x+2};\ \ x \geq -2$ is ...

(a) $f^{-1}(x) = x^2 + 2;\ \ x \geq 0$

(b) $f^{-1}(x) = x^2 - 2;\ \ x \geq 0$

(c) $f^{-1}(x) = x^2 + 2;\ \ $ all real x

(d) $f^{-1}(x)$ does not exist

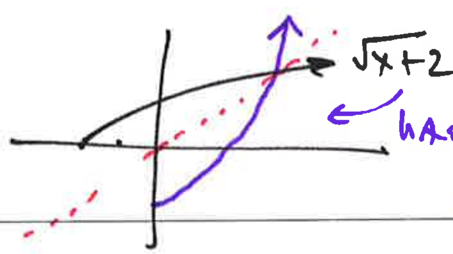
(e) $f^{-1}(x) = x^2 - 2;\ \ $ all real x

let $x = \sqrt{y+2}$ then solve for y.

$x^2 = y + 2$

$x^2 - 2 = y$ could be b or e

But.



← has this inverse graph which is x^2-2 only on $[0,\infty)$.

6. To obtain the graph of $y = \sin(x+3) - 2$, we must shift the graph of $y = \sin(x)$

(a) 3 units up and 2 units to the right

(b) 3 units to the right and 2 units down

(c) 3 units to the right and 2 units up

(d) 3 units to the left and 2 units down

(e) 3 units up and 2 units to the left

7. Let $f(x) = \begin{cases} -x-1 & \text{, if } x < 0 \\ 3 & \text{, if } x = 0 \\ 3x-1 & \text{, if } 0 < x < 1 \\ -2x+5 & \text{, if } x \geq 1 \end{cases}$.

Which of the following statements is correct about the function $f(x)$?

(a) The function $f(x)$ is continuous at $x = 1$. False $3(1)-1 \neq -2(1)+5$

(b) The function $f(x)$ is discontinuous at $x = 0$. True $-0-1 \neq 3$ and $3 \neq 3(0)-1$

(c) $\lim_{x \to 0} f(x) = 3$ False $\lim_{x \to 0} f(x) = -1$

(d) $f(1) = 2$ False $f(1) = -2(1)+5 = 3$

(e) The function $f(x)$ has a vertical asymptote at $x = 1$.
False. There are none! Nothing in any denominators to be 0!

8. What are the horizontal asymptote(s) of the function $f(x) = \dfrac{4x^2 - 7}{8x^2 + 5x + 2}$?

(a) $y = \frac{1}{2}$

(b) $y = \frac{1}{2}, y = -\frac{1}{2}$

(c) $y = -\frac{1}{2}$

(d) $y = 0$

(e) no horizontal asymptotes

$f(x) \xrightarrow{x \to \pm\infty}$

$\dfrac{4x^2}{8x^2} = 1/2$

9. The solutions of the equation $\sqrt{2}\sin x + 1 = 0$ are

(a) $x = \frac{5\pi}{4} + 2n\pi$

(b) $x = \frac{7\pi}{4} + 2n\pi$

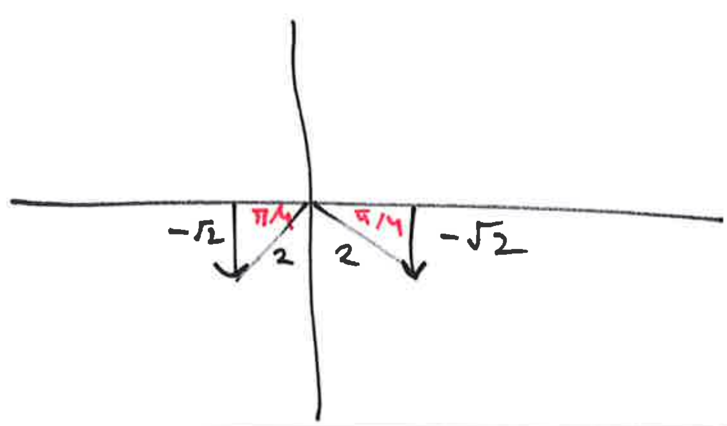
(c) $x = \frac{5\pi}{4} + n\pi$ and $x = \frac{7\pi}{4} + n\pi$

(d) $x = \frac{5\pi}{4} - n\pi$ and $x = \frac{7\pi}{4} - n\pi$

(e) $x = \frac{5\pi}{4} + 2n\pi$ and $x = \frac{7\pi}{4} + 2n\pi$

with $n = 0, \pm 1, \pm 2, \ldots$

$\sin x = -\dfrac{1}{\sqrt{2}} = -\dfrac{\sqrt{2}}{2}$



$\pi + \pi/4 = 5\pi/4$ $\quad -\pi/4 + 2\pi = 7\pi/4$

10. The range of the function $f(x) = \sin^{-1} x$ is

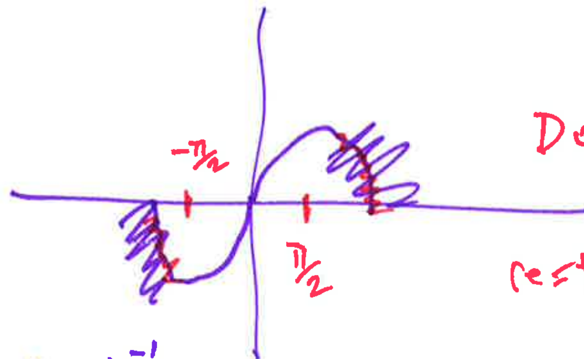
(a) $[-1, 1]$

(b) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

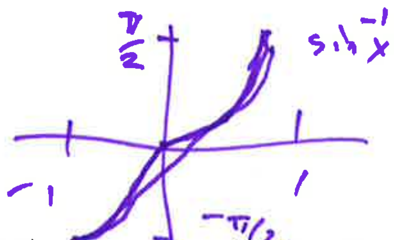
(c) $[0, \pi]$

(d) $(-\infty, \infty)$

(e) $[-\pi, \pi]$



Domain of $\sin x$ is restricted to $[-\frac{\pi}{2}, \frac{\pi}{2}]$ so that $\sin^{-1}(x)$ is a function so range is $[-\frac{\pi}{2}, \frac{\pi}{2}]$ for inverse of $\sin x$.



11. Express 2^{x+4} as an exponential function with base e.

(a) $2^{x+4} = \dfrac{e^{x+4}}{\ln(2)}$

(b) $2^{x+4} = e^{x+4}\ln(2)$

(c) $2^{x+4} = 4e^{x\ln(2)}$

(d) $2^{x+4} = e^{(x+4)\ln(2)}$

(e) $2^{x+4} = 32e^{x\ln(2)}$

$$e^{\ln \star} = \star$$

rules of log's.

$$2^{x+4} = e^{\ln 2^{(x+4)}} = e^{(x+4)\ln 2}$$

$$\ln a + \ln b = \ln a \cdot b$$

$$\ln a - \ln b = \ln \frac{a}{b}$$

$$\ln a^r = r \ln a$$

12. Find $\lim_{x \to -\infty} (2x^{-8} + 4x^3)$

(a) 2

(b) 4

(c) 0

(d) ∞

(e) $-\infty$

$\frac{2}{x^8} \to 0 \quad + 4x^3 \to -\infty$

↑ will be negative since 3 is odd.

$x \to -\infty$

Free response questions

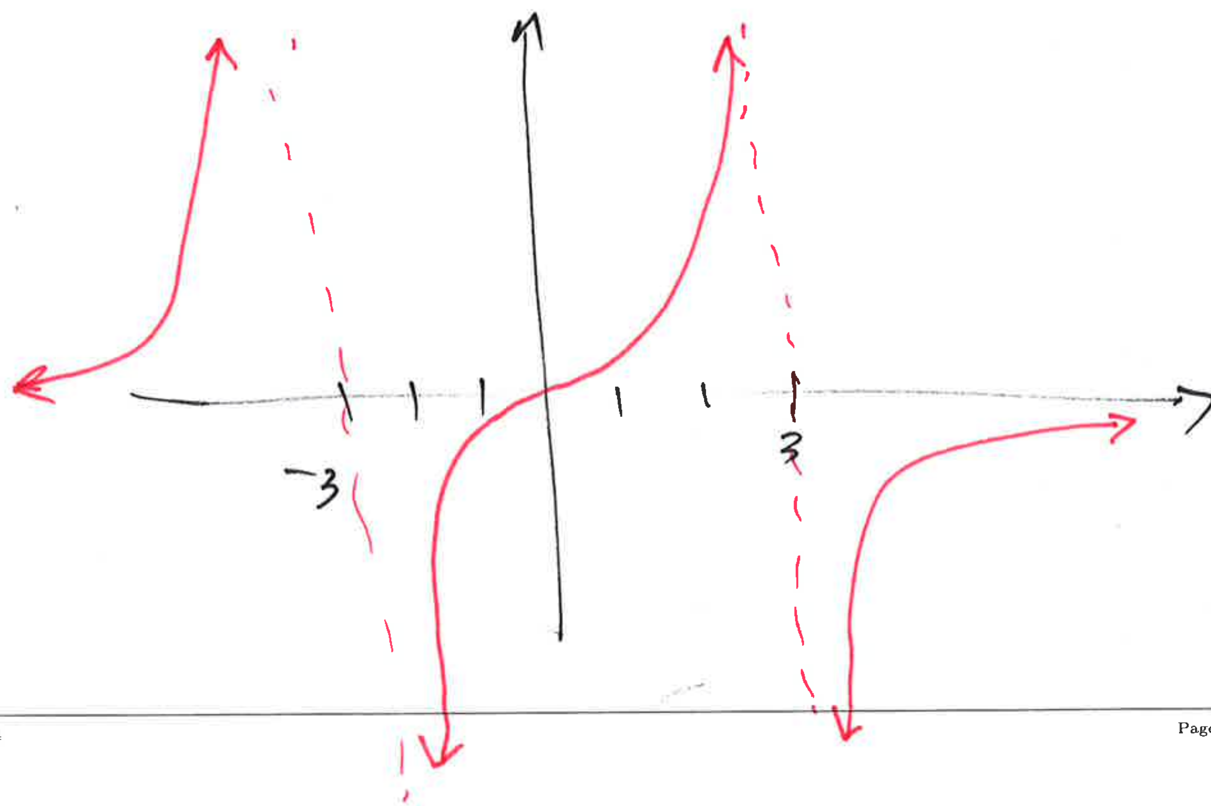
13. For what values of x is the function $f(x) = \dfrac{x}{9-x^2}$ continuous?

$$\frac{x}{(3-x)(3+x)}$$

Notice $y=0$ is horizontal asymptote!

This function has vert. Asymptotes at $x=3, -3$. No holes since nothing cancels.

So answer is $(-\infty,-3)\cup(-3,3)\cup(3,\infty)$



14. Consider is the position function $s(t) = -16t^2 + 100t$. Find the average velocity between $t = 2$ and $t = 3$.

$$\frac{s(3) - s(2)}{3-2} = \bar{s} \text{ Avg.}$$

$$s(3) = -16(9) + 100(3) = -144 + 300 = 156$$

$$s(2) = -16(4) + 100(2) = -64 + 200 = 136$$

$$\frac{156-136}{1} = \boxed{20}$$ ✓

extension: Suppose you had to find instantaneous change at $t=2$, then

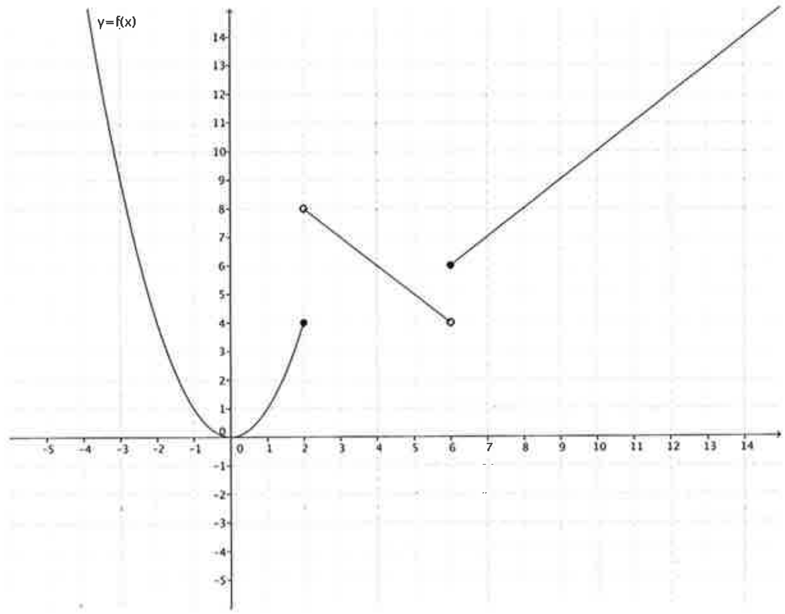
$$\lim_{h \to 0} \frac{s(2+h) - s(2)}{h} = f'(2) \text{ Derivative!}$$

$$\lim_{h \to 0} \frac{-16(2+h)^2 + 100(2+h) - (-16(2)^2 + 100(2))}{h}$$

simplifies to $\frac{-64h + 100h - 16h^2}{h} = \frac{36h - 16h^2}{h}$

$= \boxed{36 - 16h}$ as $h \to 0$ → 36 ✓

15. Consider the function represented by the graph below



Find (if it exists, otherwise write DNE)

(a) $\lim_{x \to 2^+} f(x)$ = 8

(b) $\lim_{x \to 2} f(x)$ DNE

(c) $\lim_{x \to 6^-} f(x)$ 4

(d) $\lim_{x \to 0} f(x)$ 0

(e) $f(6)$ 6